DAMAGE ASSESSMENT USING HYPERCHAOTIC EXCITATION AND STATE-SPACE GEOMETRY CHANGES

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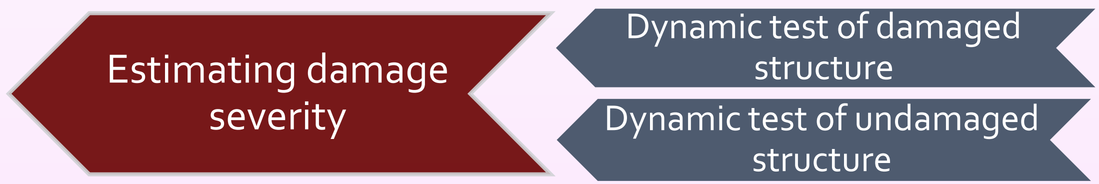
Introduction
Vibration-Based Damage Assessment

Vibration based damage assessment

- **Direct or forward approach:**

- **Inverse approach:**



Damage-sensitive features

Any changes in some characteristic 'features' of the structural dynamic response serve as indicators of damage.

Features based on time series analysis

Transformation to a state space geometric domain, has long been of the most interest in nonlinear dynamics.

Introduction
Chaotic Interrogation

Chaotic Interrogation Technique [Todd *et al* 2001]

- Uses deterministic excitation
- Uses steady state response
- Uses attractor geometry as a feature

Attractor-based features

- Correlation dimension [Logan *et al* 1996, Craig *et al* 2000 and Wang *et al* 2001]
- Local attractor variance [Todd *et al* 2001 & Nichols *et al* 2003]
- Auto-prediction error [Nichols *et al*, 2003]
- Cross-prediction error [Todd *et al*, 2004].
- Continuity [Nichols *et al* 2004].
- Chaotic amplification of attractor distortion (CAAD) [Moniz *et al* 2005].
- Transfer entropy [Todd *et al* 2005].
- Generalized Interdependence [Overbey et al 2008].

Invariants of a non-linear system

- **Lyapunov exponent:** reflects the sensitivity of the system to perturbation.
- **Fractal dimension**: measures how an attractor's geometry varies over several orders of length scale.

Chaotic excitation

- Broadband in the frequency domain.
- Deterministic and low dimensional (as low as three-dimensional).
- Extremely sensitive to small changes in system parameters (having a positive Lyapunov exponent).

Why a Hyperchaotic excitation ?

Having all the above properties, a hyperchaotic excitation has two distinguishing properties:

- More sensitive to small changes in system parameters(having more than one positive Lyapunov exponent).
- Still low dimensional (as low as four-dimensional)

Methodology
The Structure as a Filter

Linear structure forced by a nonlinear oscillator

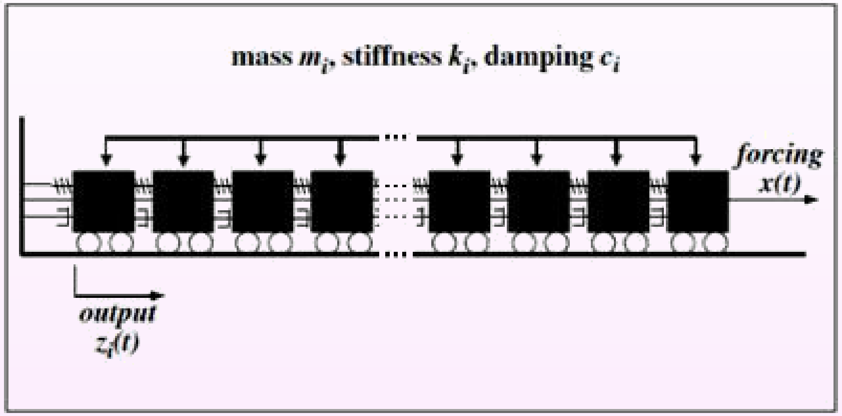
$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x})$$
$$\dot{\mathbf{z}} = \mathbf{A}\mathbf{z}(t) + \mathbf{B}\mathbf{x}(t)$$

$\mathbf{F}(\mathrm{x})$: Nonlinear vector field.

$\mathbf{x}(\mathrm{t})$: state-vector of a chaotic/hyperchaotic oscillator.

$\mathbf{A}$: Specifications of the structure (filter)

$\mathbf{B}$: Coupling matrix



Lyapunov spectrum of the response

$$\left.\begin{aligned}\lambda_i^C &: i = 1, \ldots, M \\ \lambda_j^L &: j = 1, \ldots, N\end{aligned}\right\} \Rightarrow \lambda_k^S, \qquad \lambda_1^S > \lambda_2^S > \cdots > \lambda_{N+M}^S$$

$\lambda^C{}_i$: LE's of the *M*-dimensional chaotic/hyperchaotic signal.

$\lambda^L{}_j$: LE's of the *N*-dimensional linear filter.

$\lambda^S{}_k$: LE's of the *N*+*M*-dimensional system.

Fractal Dimension of the response (Kaplan-Yorke conjecture)

$$D_L = k + \frac{\sum_{r=1}^{k} \lambda_r}{-\lambda_{k+1}}$$

k : maximum number of LE's which may be added together before the sum becomes negative

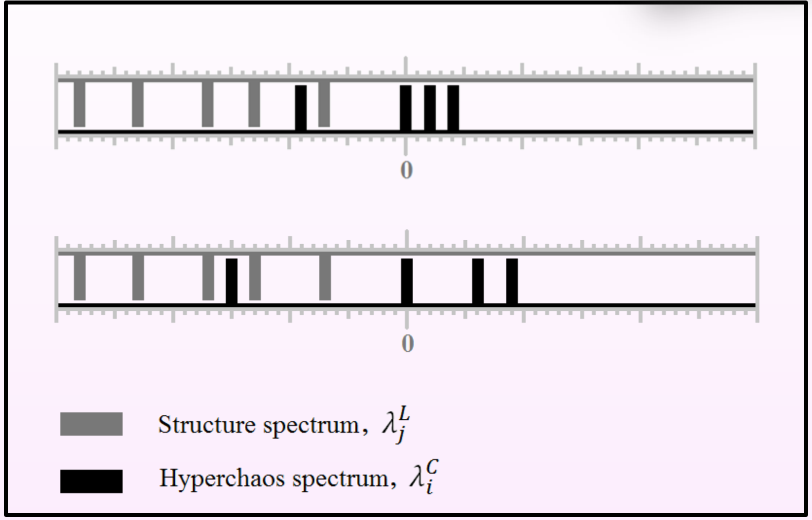
D_{L} : Lyapunov dimension(which is close, if not equal, to the fractal dimension)

Tuning criteria

$$|\lambda_M^C| > |\lambda_{d_o}^L|$$

$$\sum_{r=1}^{d_o} |\lambda_r^L| > \sum_{r=1}^{p} |\lambda_r^C| > \sum_{r=1}^{d_o-1} |\lambda_r^L|$$

- d_o : Degree of overlap
- p : Number of positive LE's of the hyperchaotic spectrum
- λ^C_i : LE's of the *M*-dimensional chaotic/hyperchaotic signal.
- λ^L_j : LE's of the *N*-dimensional linear filter.



A unit (upper) and a double (lower) degree of overlap

Delay reconstruction

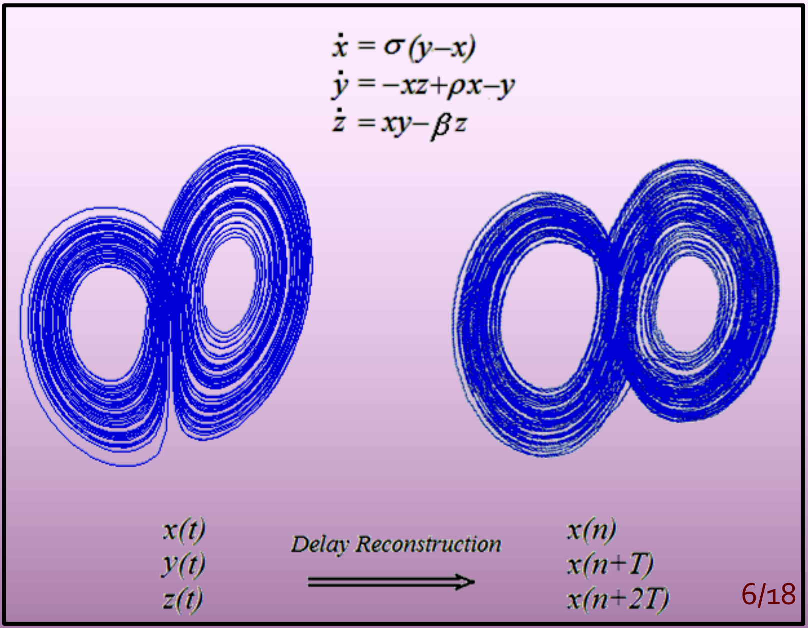
$$\mathbf{X}(n) = \begin{Bmatrix} x(n) \\ x(n+T) \\ \vdots \\ x(n+(m-1)T) \end{Bmatrix}$$

Choice of delay time, T

- Autocorrelation function
- Average mutual information function(AMI)
- Auto-covariance function

Choice of embedding dimension, m

- False nearest neighbor (FNN)
- Singular system analysis



Methodology
Average Local Attractor Variance Ratio (ALAVR) as a Feature

Algorithm to calculate (ALAVR)

- Reconstruct both the input $x(t)$ and the output attractor $z(t)$ using delay reconstruction. e.g. for $m = 2$:

$$\mathbf{X}(n) = \big(x(n), x(n+T)\big) \qquad \mathbf{Z}(n) = \big(z(n), z(n+T)\big)$$

- Select a set of N_f randomly chosen fiducial trajectory, $\boldsymbol{X}_f(n)$, on the driving attractor.
- Select a number of N_b nearest neighbors with time indices, t_j, $j = 1, \ldots, N_b$ for each fiducial point.
- Exclude points within some number of time-steps, h, (Theiler window).

$$\mathbf{X}(t_j)_n \ , \quad \mathbf{Z}(t_j)_n \quad , (n-h) \geq t_j \geq (n+h)$$

- Select the same neighborhood (same indices t_j) also from the output attractor :
- Calculate the total variance for a each of the neighborhoods:

$$var\big(Z(n)+Z(n+T)\big) = var\big(Z(n)\big) + var\big(Z(n+T)\big) + 2Cov\big(Z(n), Z(n+T)\big)$$

- Calculate the ratio R(n) ,

$$R(n) = \frac{Var\big(\mathbf{X}(t_j)_n\big)}{Var\big(\mathbf{Z}(t_j)_n\big)}$$

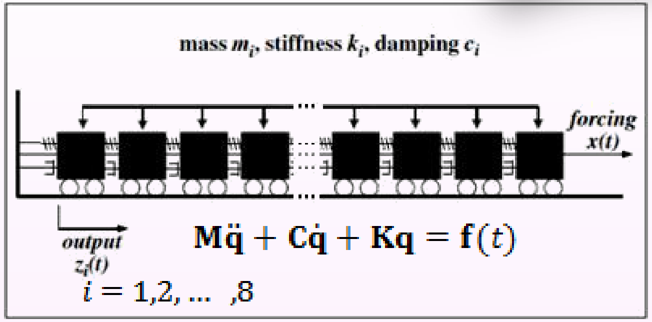
- Calculate the (ALAVR) By averaging this ratio over N_f neighborhoods:

$$\Lambda = \frac{1}{N_f}\sum_{n}^{N_f} R(n)$$

- normalize the ALAVR computed at each damage level by using the zero-damage value:

$$\alpha = \frac{|\Lambda - \Lambda^*|}{\Lambda^*}$$

Hyperchaotic Interrogation of an 8-DoF System
Modeling



Linear 8-DoF structure

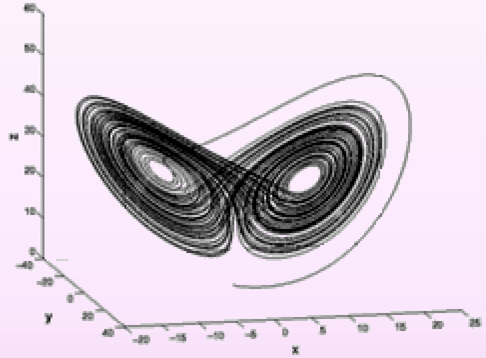
$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x})$$
$$\dot{\mathbf{z}} = \mathbf{A}\mathbf{z}(t) + \mathbf{B}\mathbf{x}(t)$$

where $\mathbf{z} = \begin{bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{bmatrix}_{2n\times 1}$ and $\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}_{2n\times 2n}$

Parameters of an 8-DoF system: $m_i = 0.01,\quad k_i = 2.0,\quad c_i = 0.075$

Lyapunov spectrum of the structure: $\lambda_j^L = -0.127, -1.123, -2.980, -5.447, -8.192, -10.843, -13.042, -14.493$

Lorenz chaotic oscillator

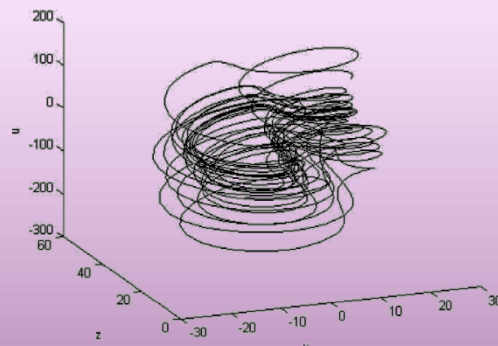


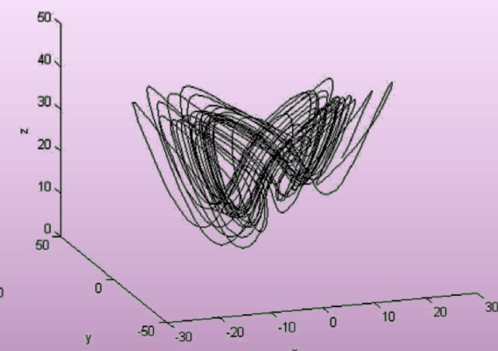
$$\dot{x}_1 = \sigma(x_2 - x_1)\delta$$
$$\dot{x}_2 = (rx_1 - x_2 - x_1x_3)\delta$$
$$\dot{x}_3 = (x_1x_2 - bx_3)\delta$$

Lyapunov spectrum: $\lambda_{i=1,2,3}^C = 0.9056, 0.000, -14.5723$

Parameters: $\sigma = 10$, $r = 28$, $b = 8/3$, δ : bandwidth control parameter

Lorenz hyperchaotic oscillator





$$\dot{x}_1 = (\sigma(x_2 - x_1) + x_4)\delta$$
$$\dot{x}_2 = (rx_1 - x_2 - x_1x_3)\delta$$
$$\dot{x}_3 = (x_1x_2 - bx_3)\delta$$
$$\dot{x}_4 = (dx_4 - x_1x_3)\delta$$

Lyapunov spectrum: $\lambda_{i=1,2,3,4}^C = 0.39854, 0.24805, 0.0000, -12.913$

Parameters: $\sigma = 10$, $r = 28$, $b = 8/3$, $d = 1.3$, δ : bandwidth control parameter

Hyperchaotic Interrogation of an 8-DoF System

Tuning the Excitation

Tuning criteria for hyperchaotic Lorenz excitation

d_o=1 → 0.01< δ <0.196 d_o=2 → 0.196< δ <1.933

d_o	δ	D_L	$\sum_{r=1}^{d_o}\lvert\lambda_r^L\rvert$	$\sum_{r=1}^{p}\lambda_r^C$	$\sum_{r=1}^{d_o-1}\lvert\lambda_r^L\rvert$	$\lvert\lambda_{d_o}^L\rvert$	$\lvert\lambda_M^C\rvert$	$\lvert\lambda_{d_o+1}^L\rvert$
1	0.01	3.050	0.127	0.129	1.123	---	0.006	0.127
1	0.05	3.251	0.127	0.645	1.123	---	0.032	0.127
1	0.085	3.432	0.127	1.097	1.123	---	0.054	0.127
1	0.095	3.483	0.127	1.226	1.123	---	0.061	0.127
1	0.170	3.865	0.127	2.195	1.123	---	0.109	0.127
1	0.196	3.997	0.127	2.530	1.123	---	0.126	0.127
2	0.21	4.007	1.123	2.71	2.980	0.127	0.135	1.25
2	0.23	4.019	1.123	2.969	2.980	0.127	0.148	1.25
2	0.25	4.093	1.123	3.22	2.980	0.127	0.161	1.25
2	0.4	4.117	1.123	5.165	2.980	0.127	0.258	1.25
2	1.933	4.999	1.123	24.96	2.980	0.127	1.249	1.25

Tuning criteria for chaotic Lorenz excitation

d_o=1 → 0.01< δ <0.14 d_o=2 → 0.140< δ <1.38

d_o	δ	D_L	$\sum_{r=1}^{d_o}\lvert\lambda_r^L\rvert$	λ_1^C	$\sum_{r=1}^{d_o-1}\lvert\lambda_r^L\rvert$	$\lvert\lambda_{d_o}^L\rvert$	$\lvert\lambda_M^C\rvert$	$\lvert\lambda_{d_o+1}^L\rvert$
1	0.01	2.070	0.127	0.145	1.123	---	0.009	0.127
1	0.07	2.496	0.127	1.020	1.123	---	0.063	0.127
1	0.14	2.998	0.127	2.040	1.123	---	0.126	0.127
2	0.25	3.088	1.123	3.643	2.980	0.127	0.226	1.25
2	1.38	3.999	1.123	20.109	2.980	0.127	1.249	1.25

Hyperchaotic Interrogation of an 8-DoF System
Integration and Reconstruction

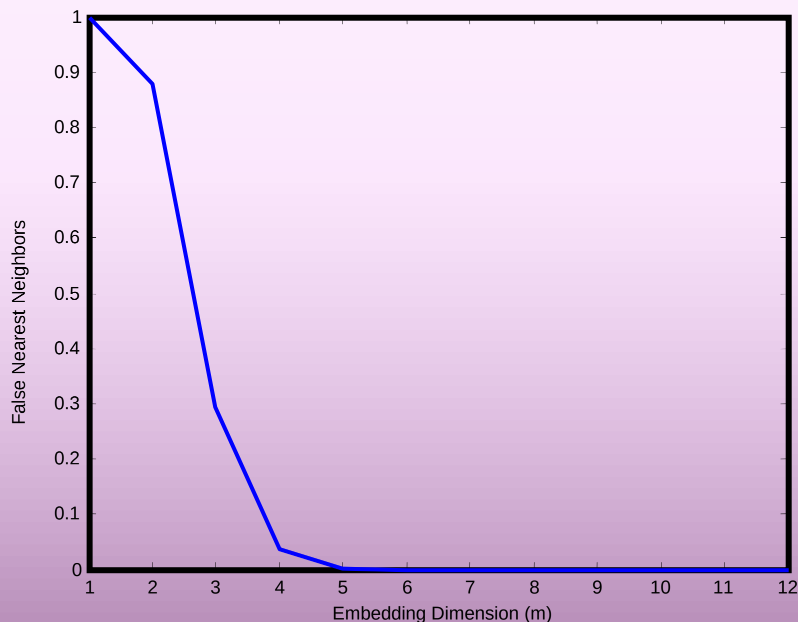
Numerical Integration

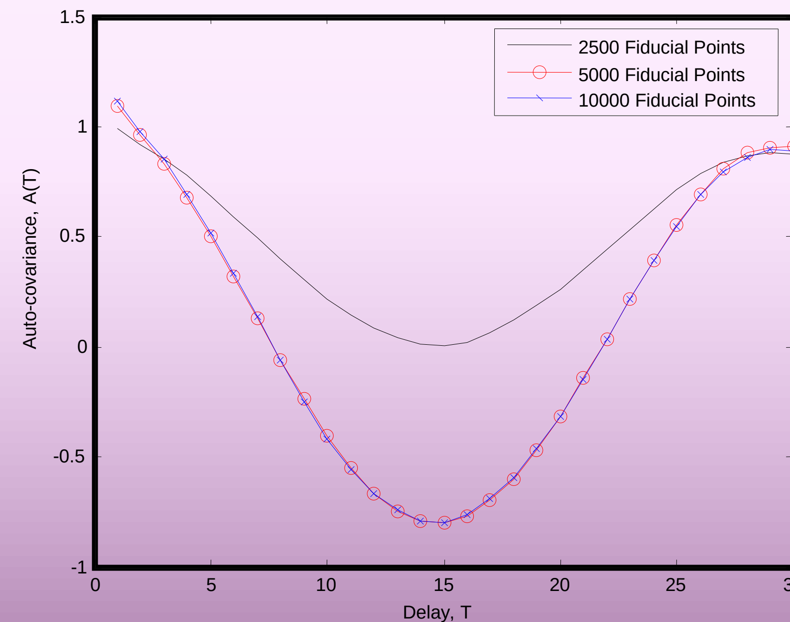
The equations of the 8-DoF system and that of the oscillator are integrated simultaneously using the 8th-order Runge-Kutta algorithm with a fixed time step of 0.0417 sec.

Delay Reconstruction

– The proper delay is selected based on auto-covariance function : $$A(T) = \frac{1}{N_f}\sum_{n=1}^{N_f} Cov\left(Z_n(T)\right)$$

– The 'False Nearest Neighbor (FNN)' is used to find the proper embedding dimension.





Hyperchaotic Interrogation of an 8-DoF System

ALAVR for a Linear Damage Scenario

ALAVR dependence on m and δ

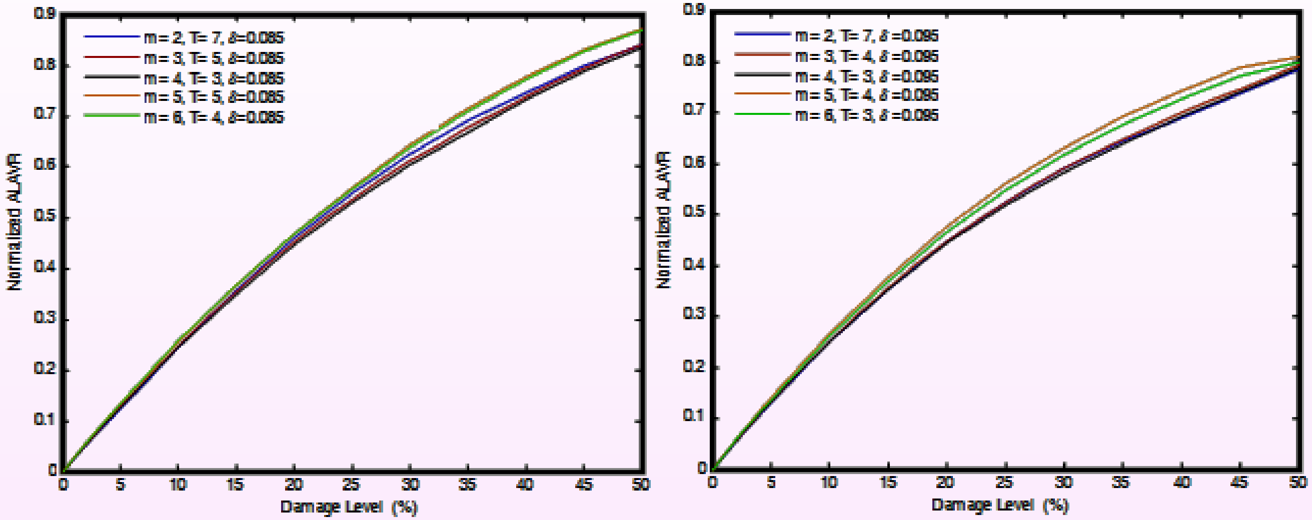


Figure 6. ALAVR obtained using different embedding dimensions($m=2,\ldots,6$) for two values of δ

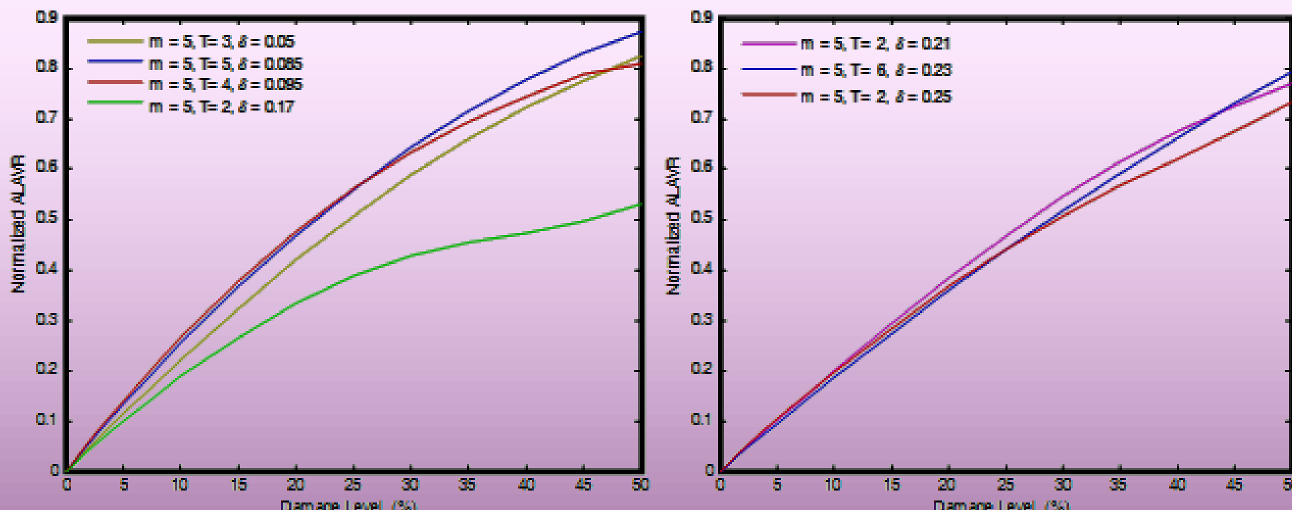
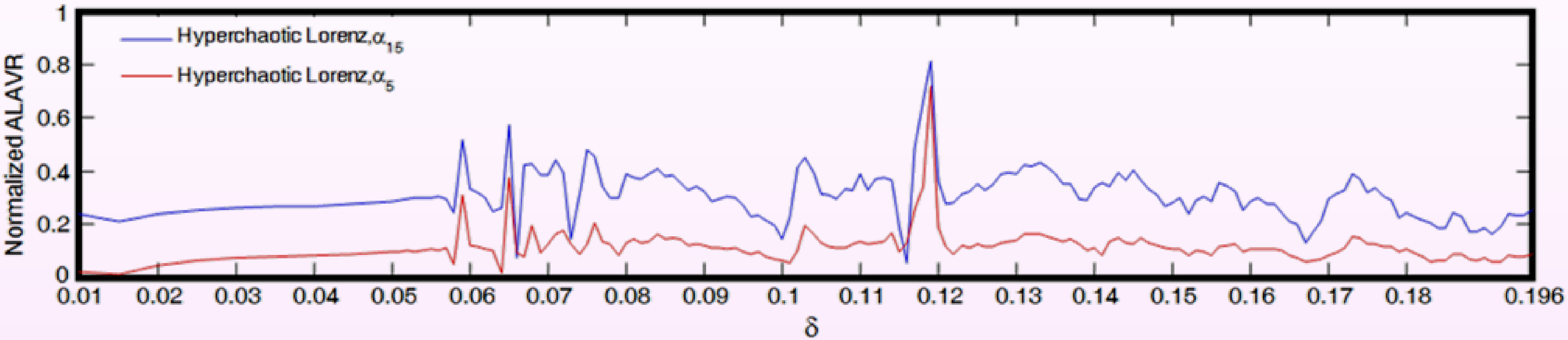


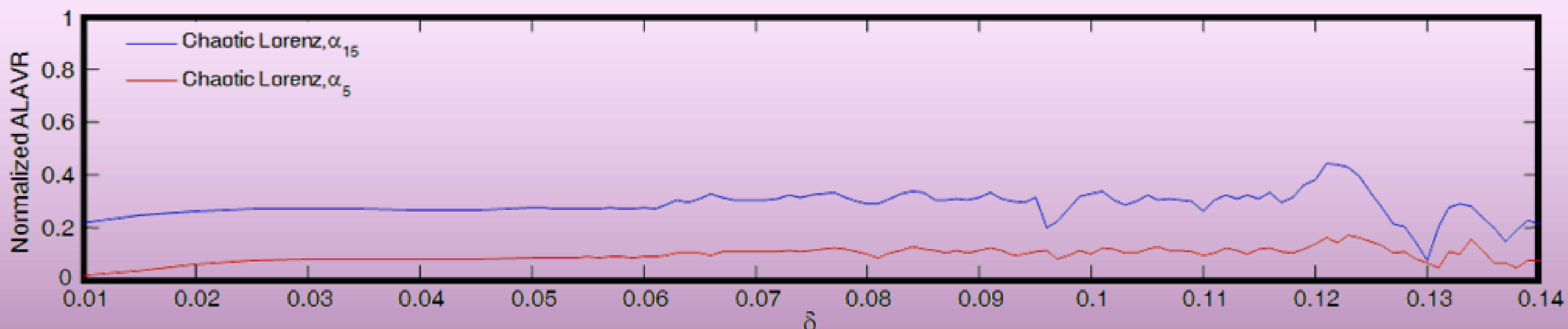
Figure 7. ALAVR obtained using different values of δ for a degree of overlap of one (left) and two (right).

Hyperchaotic Interrogation of an 8-DoF System
Parametric Analysis

Varying δ, fixed m, T based on auto-covariance



Normalized ALAVR at 5 and 15 percent of damage versus bandwidth control parameter for Hyperchaotic Lorenz excitation with m=2 and delay based on auto-covariance function.



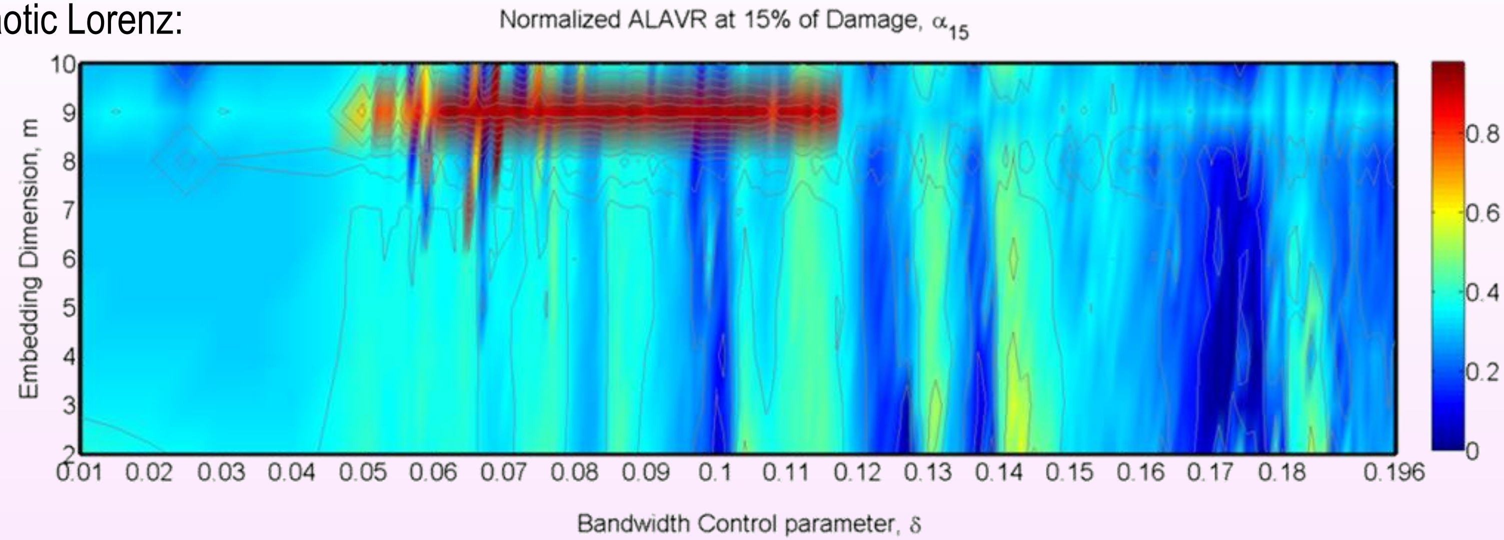
Normalized ALAVR at 5 and 15 percent of damage versus bandwidth control parameter for chaotic Lorenz excitation with m=2 and delay based on auto-covariance function.

Hyperchaotic Interrogation of an 8-DoF System

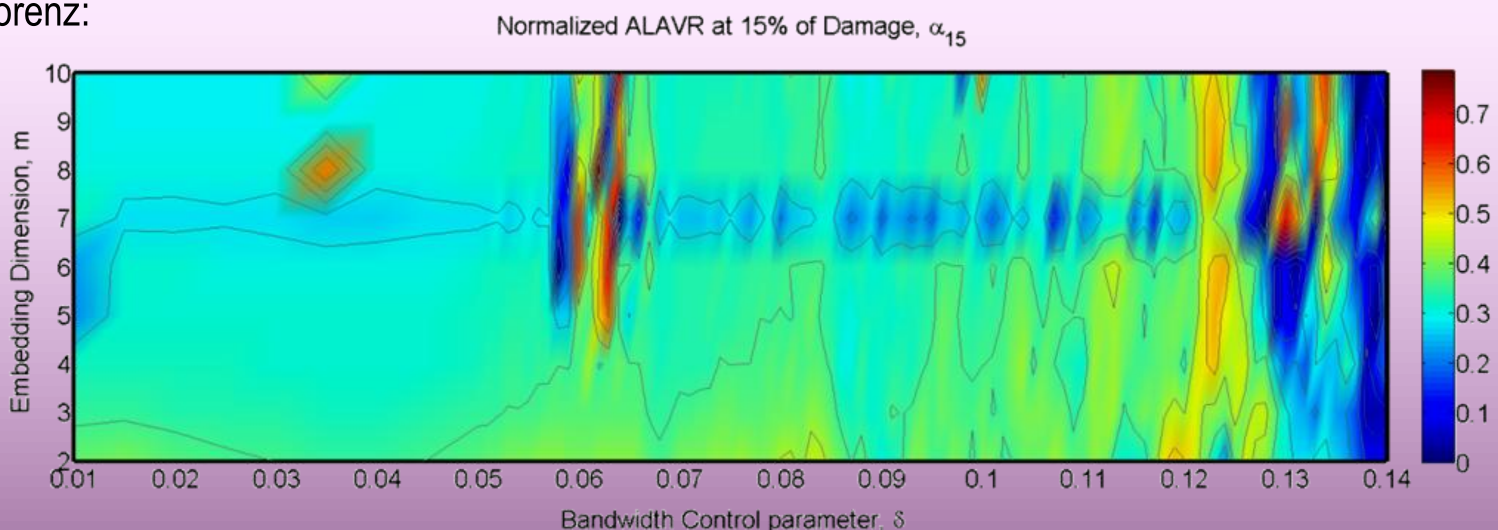
Parametric Analysis (continued)

Varying δ, varying m, unit delay

Hyperchaotic Lorenz:



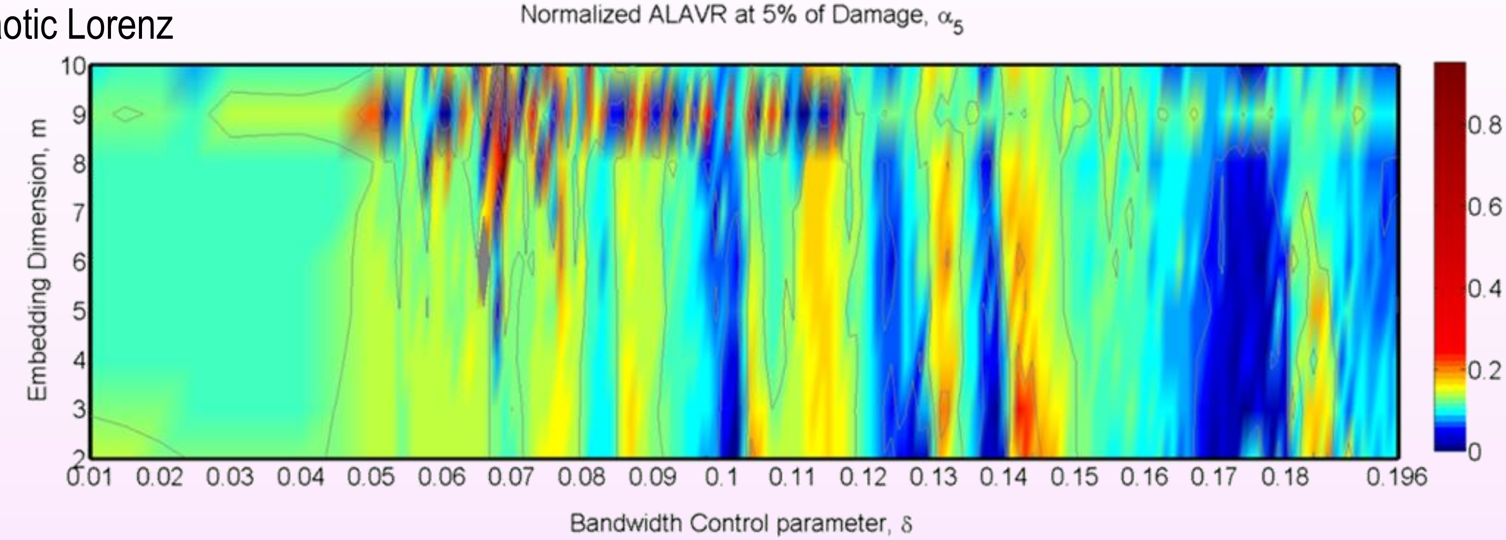
Chaotic Lorenz:



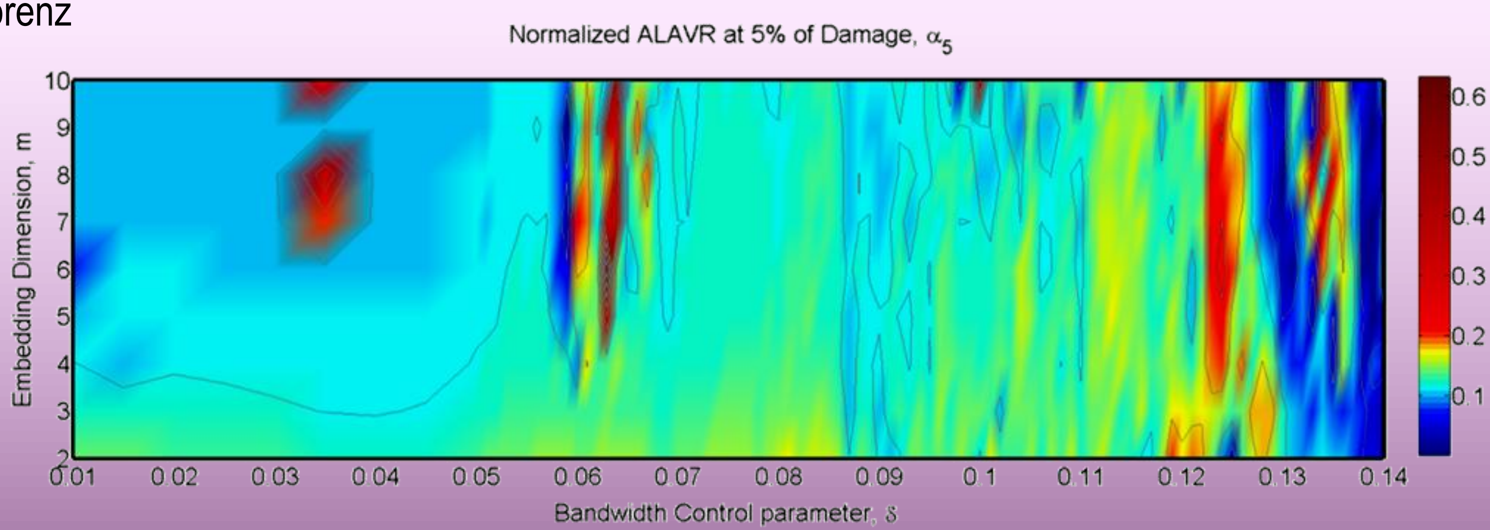
Hyperchaotic Interrogation of an 8-DoF System
Parametric Analysis (continued)

Varying δ, varying m, unit delay

Hyperchaotic Lorenz



Chaotic Lorenz



Hyperchaotic Interrogation of an 8-DoF System

Sensitivity Comparison

Most sensitive case:

δ: Based on parametric analysis

m: Based on parametric analysis

T: Based on Auto-Covariance

d_o: Unit

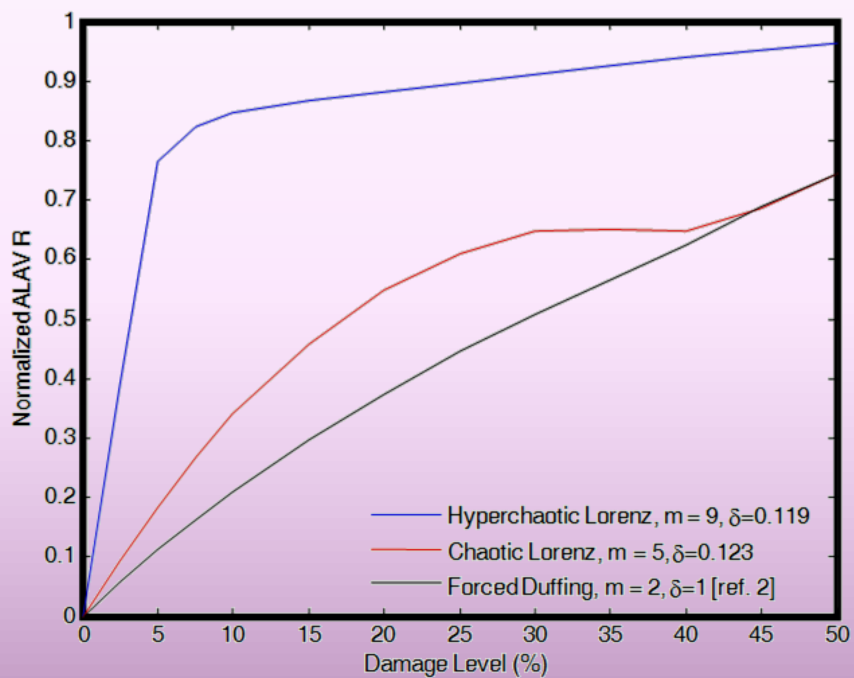
An arbitrary case:

δ: Arbitrary

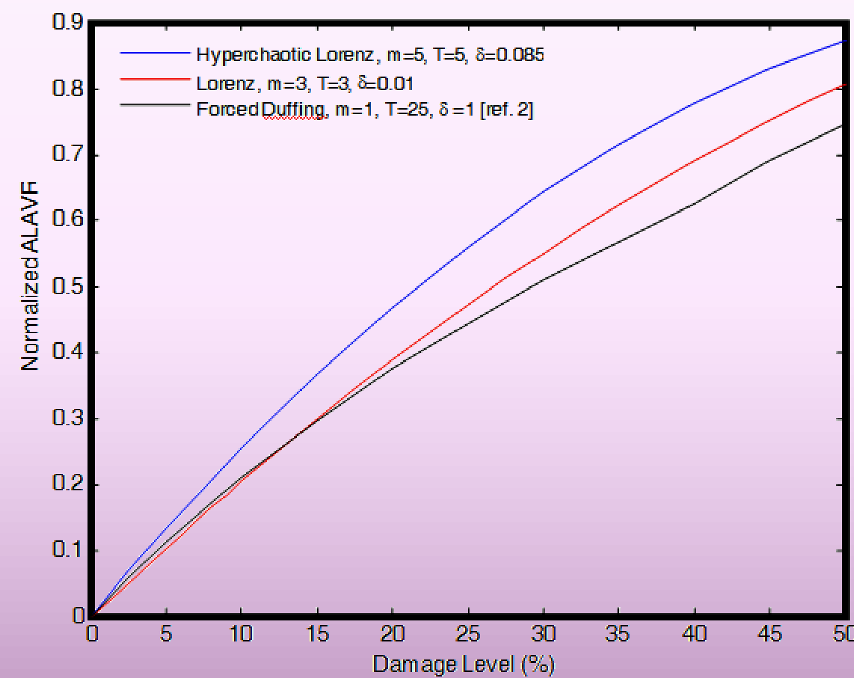
m: Based on FNN

T: Based on Auto-Covariance

d_o: Unit



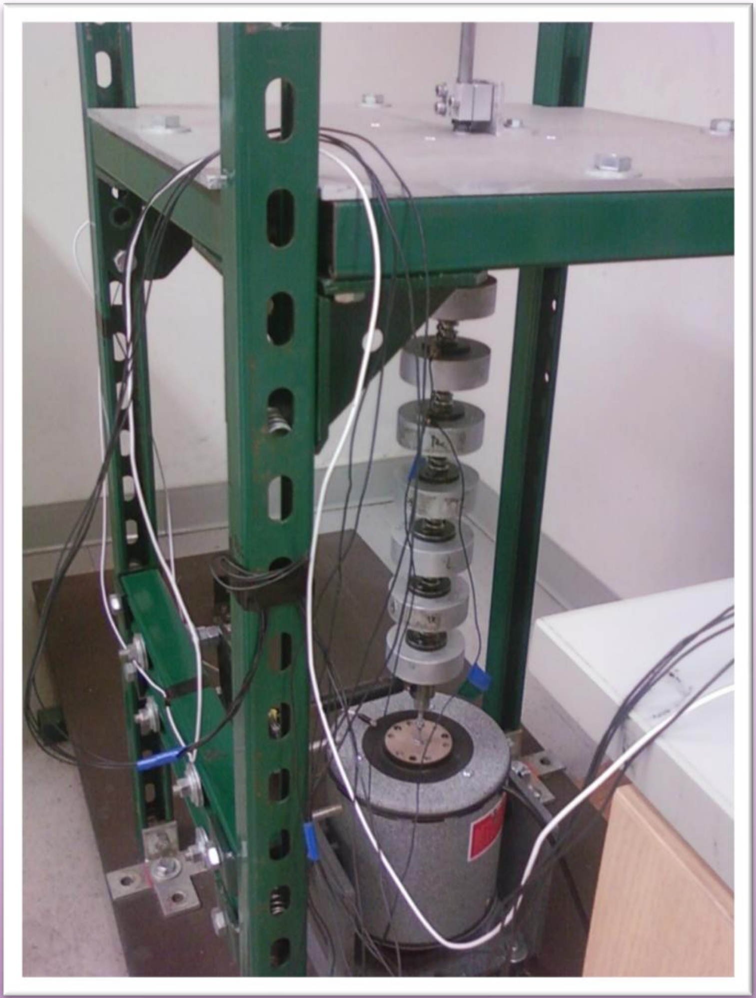
Comparing the highest sensitivity based on parametric analysis



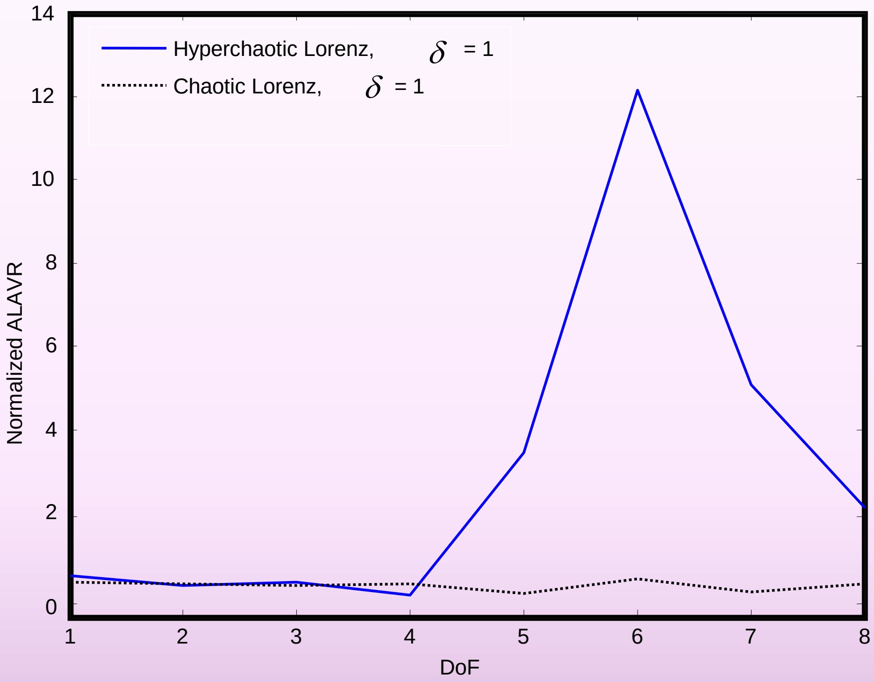
Comparing the sensitivity at some arbitrary values of δ

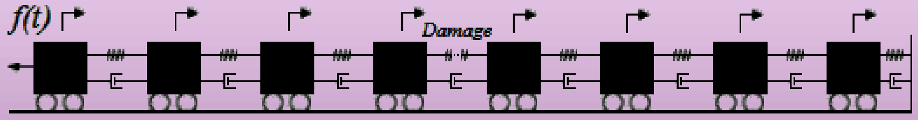
Experimental Verification

Hyperchaotic Interrogation of an 8-DoF System



Using hyperchaotic interrogation for damage detection of an 8DoF system





ALAVR Feature at each DoF of the system obtained from hyperchaotic and chaotic interrogation

Conclusion

- A deterministic hyperchaotic steady state dynamic can be applied to the structure for the sake of damage parameters identification.

- Comparisons between the geometry of a baseline attractor and a test attractor at some unknown state of health can be used to determine the severity of the damage parameters.

- **'Average local attractor variance ratio' (ALAVR), is an appropriate feature to be used for the sake of geometry comparison of attractors obtained from a hyperchaotically excited structure.**

- **A hyperchaotic signal not only has all the advantages that make a chaotic signal suitable for being used as an excitation, but also it is shown to be even more sensitive to subtle changes in damage severity as a result of having more than one positive Lyapunov exponent.**

- **In hyperchaotic interrogation technique, in addition to the overlapping of the Lyapunov spectrum of the structure and the driving attractor, choice of a proper bandwidth for the excitation signal can affect the sensitivity of the technique.**

Acknowledgement

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